Trabalho Nº 0 de TEA018 Hidrologia Ambiental

Discente 1 (GRR00000001) Discente 2 (GRR00000001)
Discente 3 (GRR00000001) Discente 4 (GRR00000001)

7 de julho de 2020

Auto-avaliação

Discente	Discussões em grupo	Formulação da solução dos problemas obrigatórios	Programação da solução dos problemas obrigatórios	Formulação da solução do material adicional	Programação da solução do material adicional	Redação e preparação do relatório
Disc 1	*	*	*			*
Disc 2	*			*		
Disc 3	*				*	*
Disc 4						

1 Questões obrigatórias

1.1 1ª Questão

Em Hidráulica, a equação de descarga para um vertedor é

$$Q = CLH^{3/2},$$

onde Q é a vazão, C é o coeficiente de descarga, L é a largura do vertedor e H é a carga sobre a soleira. No sistema britânico de unidades (Q em $\mathrm{ft}^3\ \mathrm{s}^{-1}$, L e H em ft), $C \sim 2.8$. Obtenha C no SI.

SOLUÇÃO

$$(\mathrm{m}/0.3048)^3\,\mathrm{s}^{-1} = 2.8 \times (\mathrm{m}/0.3048)^{5/2},$$
$$\mathrm{m}^3\,\mathrm{s}^{-1} = 2.8 \times \frac{(0.3048)^3}{(0.3048)^{5/2}}\mathrm{m}^{5/2};\ \Rightarrow$$
$$C = 2.8 \times 0.3048^{1/2} = 1.546.$$

Uma outra abordagem é tentar um ataque *racional* e dimensionalmente consistente para o problema. Note que a equação

$$Q = CLH^{3/2}$$

é *dimensionalmente inconsistente*: uma evidência disso é que C muda de valor quando mudamos o sistema de unidades. É relativamente fácil corrigir isso, entretanto. Na equação acima a vazão cresce linearmente com a largura da soleira; portanto, basta considerar a *vazão por unidade de largura*, $q = Q/L$. Esta por sua vez depende claramente de H. Com um pequeno esforço, notamos que o escoamento é forçado apenas pela gravidade; incluímos portanto a aceleração da gravidade g na lista de variáveis intervenientes. Temos agora 3 variáveis (q, g e H) e 2 dimensões fundamentais: o comprimento L e o tempo T. A lista de dimensões das variáveis é

$$[\![q]\!] = \mathsf{L}^2\mathsf{T}^{-1},$$
$$[\![g]\!] = \mathsf{L}\mathsf{T}^{-2},$$
$$[\![H]\!] = \mathsf{L}.$$

A matriz dimensional é

	q	g	H
L	2	1	1
T	−1	−2	0

Existe apenas um parâmetro adimensional, que tem que ser constante; portanto,

$$\frac{q}{H\sqrt{gH}} = \alpha \qquad \Rightarrow \qquad Q = \alpha\sqrt{g}LH^{3/2}.$$

Uma segunda forma de responder à questão sobre o valor de C no SI, portanto, é reconhecer que $C = \alpha\sqrt{g}$ (em qualquer sistema de unidades!), onde agora α é uma constante adimensional e *universal*. No sistema britânico, $g = 32.2\,\mathrm{ft\,s}^{-2}$; logo,

$$\alpha\sqrt{32.2} = 2.8;$$
$$\alpha = \frac{2.8}{\sqrt{32.2}} = 0.493.$$

Portanto, no SI nós revertemos o raciocínio:

$$C = \alpha\sqrt{g} = 0.493 \times \sqrt{9.81} = 1.545\ \blacksquare$$

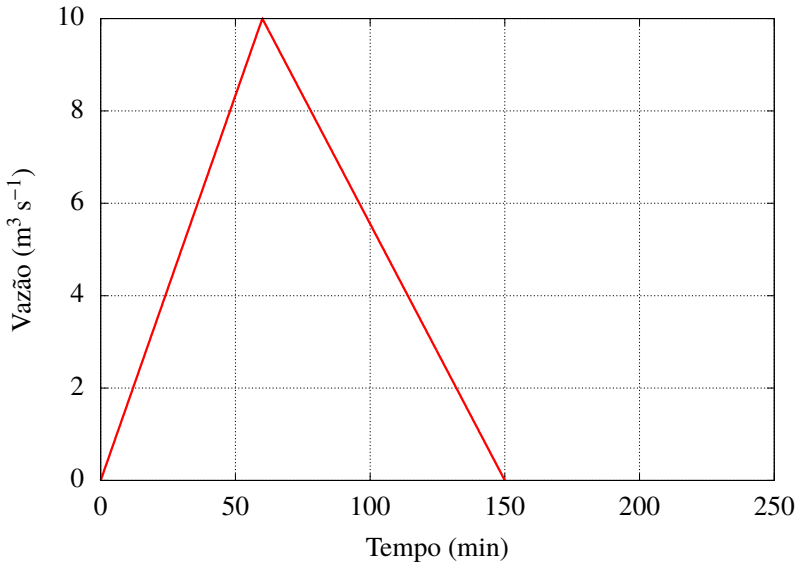

Figura 1: Cheia afluente a um reservatório de acumulação urbano.

1.2 2ª Questão

Um reservatório de acumulação de cheias urbano tem uma área horizontal $A = 100000\,\mathrm{m}^2$ e paredes verticais. O reservatório está inicialmente vazio, e recebe uma cheia $I(t)$ mostrada em vermelho na figura 1. O reservatório possui um vertedor de soleira livre e largura $L = 20\,\mathrm{m}$. Use o coeficiente C calculado acima, e obtenha a vazão efluente $O(t)$ em função do tempo, de 1 em 1 minuto. Resolva o problema usando um método de diferenças finitas *de sua escolha* para a equação de balanço hídrico do reservatório,

$$\frac{\mathrm{d}S}{\mathrm{d}t} = I(t) - O(t).$$

SOLUÇÃO

As paredes do reservatório de acumulação são verticais:

$$S = AH.$$

No instante inicial, o reservatório está vazio:

$$S(0) = H(0) = 0.$$

Isso nos dá a condição inicial do problema.

A vazão efluente é “triangular”, com duas retas cujas equações são facilmente obtidas:

$$I(t) = \begin{cases} t/360, & 0 \le t \le 3600, \\ 50/3 - t/540 & 3600 < t \le 9000. \end{cases}$$

Note que nós já convertemos as equações de o gráfico da figura 1 para t em segundos. A equação diferencial que temos que resolver é

$$\begin{aligned}
\frac{\mathrm{d}S}{\mathrm{d}t} &= I(t) - O(t), \\
S &= AH, \\
O(t) &= \alpha\sqrt{g}L[H(t)]^{3/2}, \\
\frac{\mathrm{d}[AH]}{\mathrm{d}t} + \alpha\sqrt{g}L[H(t)]^{3/2} &= I(t), \\
\frac{\mathrm{d}H}{\mathrm{d}t} + \left[\frac{\alpha\sqrt{g}L}{A}\right] H^{3/2} &= \frac{I(t)}{A}, \\
\frac{\mathrm{d}H}{\mathrm{d}t} + bH^{3/2} &= f(t), \\
b &= \frac{\alpha\sqrt{g}L}{A}, \\
f(t) &= \frac{I(t)}{A}.
\end{aligned}$$

Solução A forma mais simples de resolver numericamente a equação diferencial deste problema é utilizar um esquema de diferenças finitas explícito de 1ª ordem:

$$\begin{aligned}
\frac{H_{n+1} - H_n}{\Delta t} + bH_n^{3/2} &= f(t_n); \\
H_{n+1} - H_n + b\Delta t H_n^{3/2} &= f(t_n)\Delta t; \\
H_{n+1} &= H_n - b\Delta t H_n^{3/2} + f(t_n)\Delta t; \\
H_{n+1} &= H_n + \Delta t \left[f(t_n) - bH_n^{3/2}\right].
\end{aligned}$$

O programa de computador `rout01.py` foi escrito para resolver o problema, e é mostrado na listagem 1.

Listagem 1: `rout01.py` — Propagação de cheia com um método explícito.

```
1  #!/home/nldias/miniconda3/bin/python3
2  #-*- coding: iso-8859-1 -*-
3  #------------------------------------------------------------------------
4  # rout01: propagação de cheia em um reservatório de acumulação de
5  # cheias com um esquema explícito
6  #
7  # Nelson Luís Dias
8  # 2020-07-05T12:26:50
9  # ------------------------------------------------------------------------
10 from math import sqrt
11 # ------------------------------------------------------------------------
12 # constantes do problema
13 # ------------------------------------------------------------------------
14 A = 100000.0                  # área horizontal do reservatório
15 alfa = 0.493                  # constante universal para um vertedor
```

```
16 g = 9.81                       # aceleração da gravidade
17 L = 20.0                       # largura da soleira
18 b = alfa * sqrt(g) * L / A     # cte da eq diferencial
19 # ---------------------------------------------------------------------
20 # hidrógrafa afluente (m3/s/m2)
21 # ---------------------------------------------------------------------
22 def f(t):
23    if t <= 3600:
24       return (t/360.0)/A
25    elif t <= 9000:
26       return (50.0/3.0 - t/540.0)/A
27    else :
28       return 0.0
29    pass
30 pass
31 # ---------------------------------------------------------------------
32 # tudo pronto para resolver?
33 # ---------------------------------------------------------------------
34 fou = open('rout01.out','wt')
35 told = 0
36 Iold = 0.0
37 Hold = 0.0                      # altura inicial
38 Oold = 0.0                      # hidrógrafa efluente inicial
39 fou.write('␣␣␣Tempo␣(s)␣I(t)␣(m3/s)␣O(t)␣(m3/s)\n')
40 fou.write('␣␣␣␣%8d␣␣␣␣%8.2f␣␣␣␣%8.2f\n' % (told,Iold,Oold))
41 # ---------------------------------------------------------------------
42 # aplicação do método explícito. note que told, tnew e deltat são
43 # variáveis inteiras.
44 # ---------------------------------------------------------------------
45 deltat = 60                     # passo de tempo de um minuto
46 while told < 36000 :
47    tnew = told + deltat
48    Inew = A*f(tnew)
49    Hnew = Hold + deltat*( f(told) - b*Hold**1.5)
50    Onew = b*A*Hnew**1.5
51 # ---------------------------------------------------------------------
52 # imprime esta linha de resultados
53 # ---------------------------------------------------------------------
54    fou.write('␣␣␣␣%8.2f␣␣␣␣%8.2f␣␣␣␣%8.2f\n' % (tnew,Inew,Onew))
55    told = tnew
56    Hold = Hnew
57 pass
58 fou.close()
```

A saída da simulação é mostrada na listagem 3

Graficamente, a figura 2 mostra o resultado da simulação.

2 Material adicional

O mesmo problema também pode ser resolvido facilmente pelo método de Runge-Kutta de 4ª ordem (Dias, 2020). O programa `rout02.py` implementa essa solução alterntiva.

Inicialmente, colocamos a equação diferencial em uma forma reconhecível pelo método

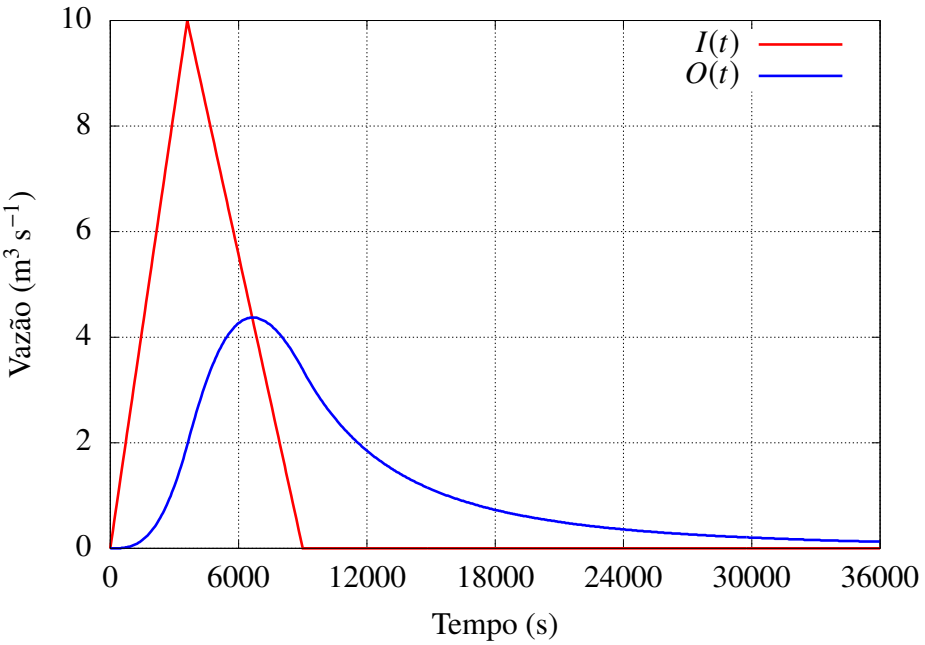

Figura 2: Simulação de um reservatório de acumulação de cheias com um método explícito.

de Runge-Kutta:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = F(t, H) = f(t) - bH^{3/2}.$$

Com isso, é muito simples implementar $F(t, H)$ (que denominamos `FF` no programa `rout02.py`). O programa completo é mostrado na listagem 2.

Listagem 2: `rout02.py` — Propagação de cheia com o método de Runge-Kutta.

```
#!/home/nldias/miniconda3/bin/python3
# -*- coding: iso-8859-1 -*-
#-----------------------------------------------------------------------
# rout02: propagação de cheia em um reservatório de acumulação de
# cheias com um método de Runge-Kutta de 4a ordem
#
# Nelson Luís Dias
# 2020-07-05T13:41:13
# -----------------------------------------------------------------------
from math import sqrt
# -----------------------------------------------------------------------
# constantes do problema
# -----------------------------------------------------------------------
A = 100000.0                  # área horizontal do reservatório
alfa = 0.493                  # constante universal para um vertedor
g = 9.81                      # aceleração da gravidade
L = 20.0                      # largura da soleira
b = alfa * sqrt(g) * L / A    # cte da eq diferencial
# -----------------------------------------------------------------------
# hidrógrafa afluente (m3/s/m2)
# -----------------------------------------------------------------------
```

```
22 def f(t):
23    if t <= 3600:
24       return (t/360.0)/A
25    elif t <= 9000:
26       return (50.0/3.0 - t/540.0)/A
27    else :
28       return 0.0
29    pass
30 pass
31 def FF(t,H):
32    return f(t) - b*H**1.5
33 pass
34 # ---------------------------------------------------------------------
35 # método de Runge-Kutta
36 # ---------------------------------------------------------------------
37 def rk4(t,H,deltat,FF):
38    '''
39    rk4 implementa um passo do método de Runge-Kutta de ordem 4
40    '''
41    k1 = deltat*FF(t,H)
42    k2 = deltat*FF(t+deltat/2,H+k1/2)
43    k3 = deltat*FF(t+deltat/2,H+k2/2)
44    k4 = deltat*FF(t+deltat,H+k3)
45    Hn = H + k1/6.0 + k2/3.0 + k3/3.0 + k4/6.0
46    return Hn
47 pass
48 # ---------------------------------------------------------------------
49 # Propaga a cheia usando o método de Runge-Kutta de ordem 4
50 # ---------------------------------------------------------------------
51 deltat = 60.0                      # passo em t
52 t = [0.0]                          # t inicial
53 H = [0.0]                          # H inicial
54 NN = int(36000.0/deltat)           # número de passos
55 for n in range(0,NN):              # loop da solução numérica
56    tn = (n+1)*deltat               # novo t
57    Hn = rk4(t[n],H[n],deltat,FF)   # novo H
58    t.append(tn)                    # adiciona t à lista
59    H.append(Hn)                    # adiciona H à lista
60 pass
61 fou = open('rout02.out','wt')
62 fou.write('␣␣␣Tempo␣(s)␣I(t)␣(m3/s)␣O(t)␣(m3/s)\n')
63 for n in range(0,NN+1):        # imprime no arquivo de saída
64    fou.write('␣␣␣␣%8d␣␣␣␣%8.2f␣␣␣␣%8.2f\n' % (t[n],A*f(t[n]),b*A*H[n]**1.5))
65 pass
66 fou.close()
```

A saída do programa é mostrada na listagem 4

Graficamente, a figura 3 mostra o resultado da simulação.

Finalmente, nós comparamos as duas soluções na figura 4. Como podemos ver, embora o método de Runge-Kutta seja teoricamente muito mais acurado, o intervalo de tempo de simulação $\Delta t = 60\,\mathrm{s}$ é suficientemente pequeno para que o método explícito produza um bom resultado, praticamente igual ao obtido pelo método de Runge-Kutta.

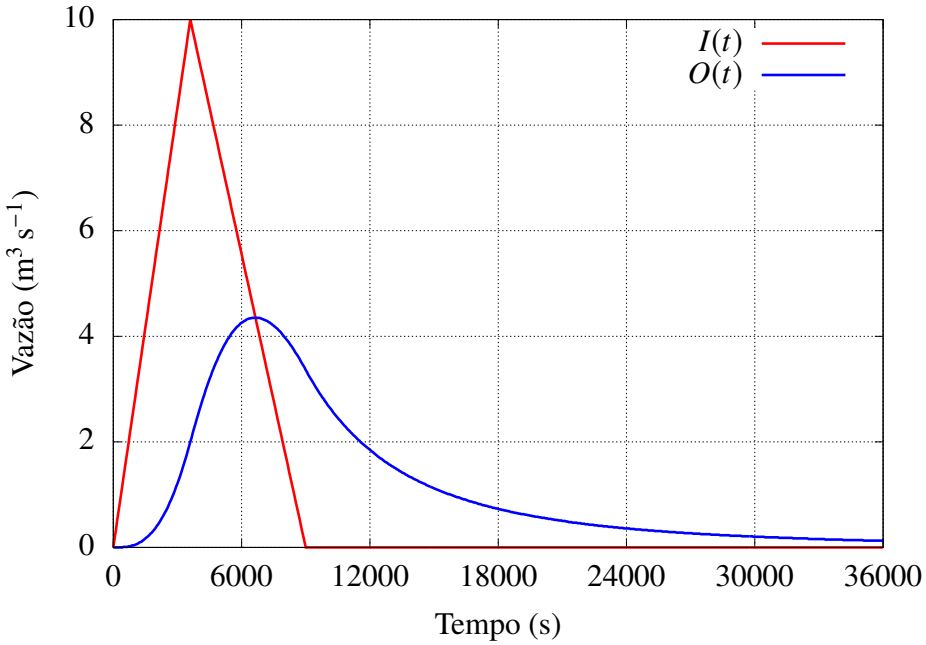

Figura 3: Simulação de um reservatório de acumulação de cheias com o método de Runge-Kutta.

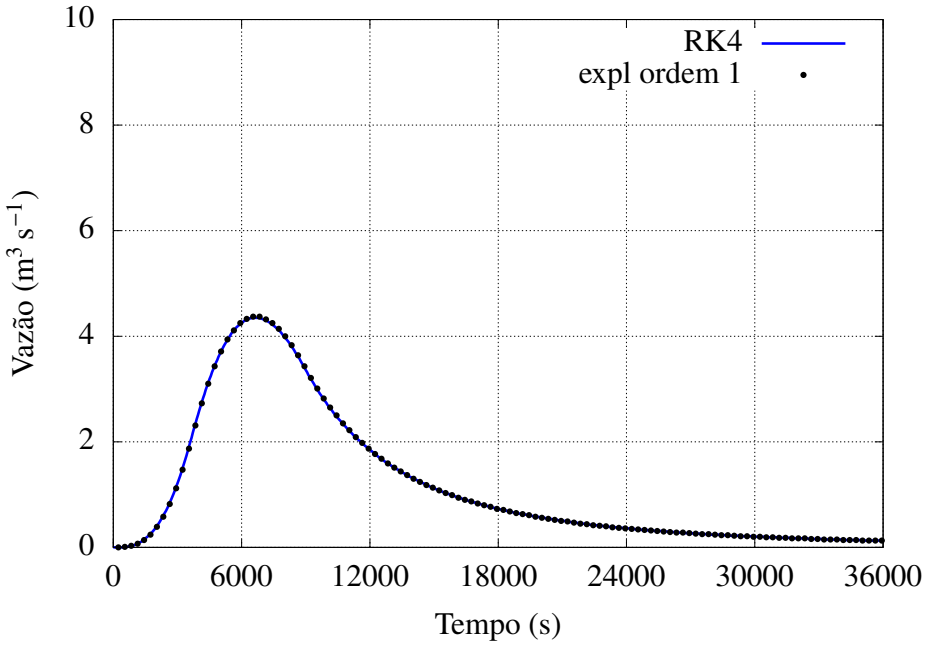

Figura 4: Comparação entre os métodos explícito (ordem 1) (pontos pretos) e de Runge-Kutta (linha azul) para a simulação de um reservatório de acumulação de cheias.

A Saídas das simulações realizadas

Listagem 3: `rout01.py` — Resultado numérico do programa `rout01.py` (arquivo `rout01.out`)

```
   Tempo (s) I(t) (m3/s) O(t) (m3/s)
           0        0.00        0.00
       60.00        0.17        0.00
      120.00        0.33        0.00
      180.00        0.50        0.00
      240.00        0.67        0.00
      300.00        0.83        0.00
      360.00        1.00        0.00
      420.00        1.17        0.00
      480.00        1.33        0.00
      540.00        1.50        0.01
      600.00        1.67        0.01
      660.00        1.83        0.01
      720.00        2.00        0.02
      780.00        2.17        0.02
      840.00        2.33        0.03
      900.00        2.50        0.03
      960.00        2.67        0.04
     1020.00        2.83        0.05
     1080.00        3.00        0.06
     1140.00        3.17        0.07
     1200.00        3.33        0.08
     1260.00        3.50        0.09
     1320.00        3.67        0.11
     1380.00        3.83        0.12
     1440.00        4.00        0.14
     1500.00        4.17        0.16
     1560.00        4.33        0.18
     1620.00        4.50        0.20
     1680.00        4.67        0.22
     1740.00        4.83        0.24
     1800.00        5.00        0.27
     1860.00        5.17        0.30
     1920.00        5.33        0.33
     1980.00        5.50        0.36
     2040.00        5.67        0.39
     2100.00        5.83        0.42
     2160.00        6.00        0.46
     2220.00        6.17        0.50
     2280.00        6.33        0.54
     2340.00        6.50        0.58
     2400.00        6.67        0.63
     2460.00        6.83        0.67
     2520.00        7.00        0.72
     2580.00        7.17        0.77
     2640.00        7.33        0.82
     2700.00        7.50        0.88
     2760.00        7.67        0.94
     2820.00        7.83        0.99
     2880.00        8.00        1.06
```

2940.00	8.17	1.12
3000.00	8.33	1.18
3060.00	8.50	1.25
3120.00	8.67	1.32
3180.00	8.83	1.39
3240.00	9.00	1.47
3300.00	9.17	1.54
3360.00	9.33	1.62
3420.00	9.50	1.70
3480.00	9.67	1.79
3540.00	9.83	1.87
3600.00	10.00	1.96
3660.00	9.89	2.05
3720.00	9.78	2.14
3780.00	9.67	2.23
3840.00	9.56	2.31
3900.00	9.44	2.40
3960.00	9.33	2.48
4020.00	9.22	2.57
4080.00	9.11	2.65
4140.00	9.00	2.73
4200.00	8.89	2.80
4260.00	8.78	2.88
4320.00	8.67	2.96
4380.00	8.56	3.03
4440.00	8.44	3.10
4500.00	8.33	3.17
4560.00	8.22	3.24
4620.00	8.11	3.30
4680.00	8.00	3.37
4740.00	7.89	3.43
4800.00	7.78	3.49
4860.00	7.67	3.54
4920.00	7.56	3.60
4980.00	7.44	3.65
5040.00	7.33	3.71
5100.00	7.22	3.76
5160.00	7.11	3.80
5220.00	7.00	3.85
5280.00	6.89	3.89
5340.00	6.78	3.94
5400.00	6.67	3.98
5460.00	6.56	4.01
5520.00	6.44	4.05
5580.00	6.33	4.08
5640.00	6.22	4.11
5700.00	6.11	4.14
5760.00	6.00	4.17
5820.00	5.89	4.20
5880.00	5.78	4.22
5940.00	5.67	4.25
6000.00	5.56	4.27
6060.00	5.44	4.28
6120.00	5.33	4.30
6180.00	5.22	4.32
6240.00	5.11	4.33

6300.00	5.00	4.34
6360.00	4.89	4.35
6420.00	4.78	4.36
6480.00	4.67	4.36
6540.00	4.56	4.37
6600.00	4.44	4.37
6660.00	4.33	4.37
6720.00	4.22	4.37
6780.00	4.11	4.37
6840.00	4.00	4.37
6900.00	3.89	4.36
6960.00	3.78	4.35
7020.00	3.67	4.34
7080.00	3.56	4.33
7140.00	3.44	4.32
7200.00	3.33	4.31
7260.00	3.22	4.30
7320.00	3.11	4.28
7380.00	3.00	4.26
7440.00	2.89	4.25
7500.00	2.78	4.23
7560.00	2.67	4.21
7620.00	2.56	4.18
7680.00	2.44	4.16
7740.00	2.33	4.14
7800.00	2.22	4.11
7860.00	2.11	4.08
7920.00	2.00	4.06
7980.00	1.89	4.03
8040.00	1.78	4.00
8100.00	1.67	3.97
8160.00	1.56	3.93
8220.00	1.44	3.90
8280.00	1.33	3.87
8340.00	1.22	3.83
8400.00	1.11	3.80
8460.00	1.00	3.76
8520.00	0.89	3.72
8580.00	0.78	3.68
8640.00	0.67	3.64
8700.00	0.56	3.60
8760.00	0.44	3.56
8820.00	0.33	3.52
8880.00	0.22	3.48
8940.00	0.11	3.43
9000.00	0.00	3.39
9060.00	0.00	3.34
9120.00	0.00	3.30
9180.00	0.00	3.25
9240.00	0.00	3.21
9300.00	0.00	3.17
9360.00	0.00	3.13
9420.00	0.00	3.09
9480.00	0.00	3.05
9540.00	0.00	3.01
9600.00	0.00	2.97

9660.00	0.00	2.93
9720.00	0.00	2.90
9780.00	0.00	2.86
9840.00	0.00	2.82
9900.00	0.00	2.79
9960.00	0.00	2.75
10020.00	0.00	2.72
10080.00	0.00	2.69
10140.00	0.00	2.65
10200.00	0.00	2.62
10260.00	0.00	2.59
10320.00	0.00	2.56
10380.00	0.00	2.53
10440.00	0.00	2.50
10500.00	0.00	2.47
10560.00	0.00	2.44
10620.00	0.00	2.41
10680.00	0.00	2.38
10740.00	0.00	2.35
10800.00	0.00	2.32
10860.00	0.00	2.30
10920.00	0.00	2.27
10980.00	0.00	2.24
11040.00	0.00	2.22
11100.00	0.00	2.19
11160.00	0.00	2.17
11220.00	0.00	2.14
11280.00	0.00	2.12
11340.00	0.00	2.09
11400.00	0.00	2.07
11460.00	0.00	2.05
11520.00	0.00	2.02
11580.00	0.00	2.00
11640.00	0.00	1.98
11700.00	0.00	1.96
11760.00	0.00	1.93
11820.00	0.00	1.91
11880.00	0.00	1.89
11940.00	0.00	1.87
12000.00	0.00	1.85
12060.00	0.00	1.83
12120.00	0.00	1.81
12180.00	0.00	1.79
12240.00	0.00	1.77
12300.00	0.00	1.75
12360.00	0.00	1.73
12420.00	0.00	1.72
12480.00	0.00	1.70
12540.00	0.00	1.68
12600.00	0.00	1.66
12660.00	0.00	1.65
12720.00	0.00	1.63
12780.00	0.00	1.61
12840.00	0.00	1.59
12900.00	0.00	1.58
12960.00	0.00	1.56

13020.00	0.00	1.55
13080.00	0.00	1.53
13140.00	0.00	1.51
13200.00	0.00	1.50
13260.00	0.00	1.48
13320.00	0.00	1.47
13380.00	0.00	1.45
13440.00	0.00	1.44
13500.00	0.00	1.43
13560.00	0.00	1.41
13620.00	0.00	1.40
13680.00	0.00	1.38
13740.00	0.00	1.37
13800.00	0.00	1.36
13860.00	0.00	1.34
13920.00	0.00	1.33
13980.00	0.00	1.32
14040.00	0.00	1.30
14100.00	0.00	1.29
14160.00	0.00	1.28
14220.00	0.00	1.27
14280.00	0.00	1.25
14340.00	0.00	1.24
14400.00	0.00	1.23
14460.00	0.00	1.22
14520.00	0.00	1.21
14580.00	0.00	1.20
14640.00	0.00	1.18
14700.00	0.00	1.17
14760.00	0.00	1.16
14820.00	0.00	1.15
14880.00	0.00	1.14
14940.00	0.00	1.13
15000.00	0.00	1.12
15060.00	0.00	1.11
15120.00	0.00	1.10
15180.00	0.00	1.09
15240.00	0.00	1.08
15300.00	0.00	1.07
15360.00	0.00	1.06
15420.00	0.00	1.05
15480.00	0.00	1.04
15540.00	0.00	1.03
15600.00	0.00	1.02
15660.00	0.00	1.01
15720.00	0.00	1.00
15780.00	0.00	1.00
15840.00	0.00	0.99
15900.00	0.00	0.98
15960.00	0.00	0.97
16020.00	0.00	0.96
16080.00	0.00	0.95
16140.00	0.00	0.94
16200.00	0.00	0.94
16260.00	0.00	0.93
16320.00	0.00	0.92

16380.00	0.00	0.91
16440.00	0.00	0.90
16500.00	0.00	0.90
16560.00	0.00	0.89
16620.00	0.00	0.88
16680.00	0.00	0.87
16740.00	0.00	0.87
16800.00	0.00	0.86
16860.00	0.00	0.85
16920.00	0.00	0.85
16980.00	0.00	0.84
17040.00	0.00	0.83
17100.00	0.00	0.82
17160.00	0.00	0.82
17220.00	0.00	0.81
17280.00	0.00	0.80
17340.00	0.00	0.80
17400.00	0.00	0.79
17460.00	0.00	0.78
17520.00	0.00	0.78
17580.00	0.00	0.77
17640.00	0.00	0.77
17700.00	0.00	0.76
17760.00	0.00	0.75
17820.00	0.00	0.75
17880.00	0.00	0.74
17940.00	0.00	0.73
18000.00	0.00	0.73
18060.00	0.00	0.72
18120.00	0.00	0.72
18180.00	0.00	0.71
18240.00	0.00	0.71
18300.00	0.00	0.70
18360.00	0.00	0.70
18420.00	0.00	0.69
18480.00	0.00	0.68
18540.00	0.00	0.68
18600.00	0.00	0.67
18660.00	0.00	0.67
18720.00	0.00	0.66
18780.00	0.00	0.66
18840.00	0.00	0.65
18900.00	0.00	0.65
18960.00	0.00	0.64
19020.00	0.00	0.64
19080.00	0.00	0.63
19140.00	0.00	0.63
19200.00	0.00	0.62
19260.00	0.00	0.62
19320.00	0.00	0.61
19380.00	0.00	0.61
19440.00	0.00	0.61
19500.00	0.00	0.60
19560.00	0.00	0.60
19620.00	0.00	0.59
19680.00	0.00	0.59

19740.00	0.00	0.58
19800.00	0.00	0.58
19860.00	0.00	0.57
19920.00	0.00	0.57
19980.00	0.00	0.57
20040.00	0.00	0.56
20100.00	0.00	0.56
20160.00	0.00	0.55
20220.00	0.00	0.55
20280.00	0.00	0.55
20340.00	0.00	0.54
20400.00	0.00	0.54
20460.00	0.00	0.53
20520.00	0.00	0.53
20580.00	0.00	0.53
20640.00	0.00	0.52
20700.00	0.00	0.52
20760.00	0.00	0.52
20820.00	0.00	0.51
20880.00	0.00	0.51
20940.00	0.00	0.50
21000.00	0.00	0.50
21060.00	0.00	0.50
21120.00	0.00	0.49
21180.00	0.00	0.49
21240.00	0.00	0.49
21300.00	0.00	0.48
21360.00	0.00	0.48
21420.00	0.00	0.48
21480.00	0.00	0.47
21540.00	0.00	0.47
21600.00	0.00	0.47
21660.00	0.00	0.46
21720.00	0.00	0.46
21780.00	0.00	0.46
21840.00	0.00	0.45
21900.00	0.00	0.45
21960.00	0.00	0.45
22020.00	0.00	0.45
22080.00	0.00	0.44
22140.00	0.00	0.44
22200.00	0.00	0.44
22260.00	0.00	0.43
22320.00	0.00	0.43
22380.00	0.00	0.43
22440.00	0.00	0.42
22500.00	0.00	0.42
22560.00	0.00	0.42
22620.00	0.00	0.42
22680.00	0.00	0.41
22740.00	0.00	0.41
22800.00	0.00	0.41
22860.00	0.00	0.41
22920.00	0.00	0.40
22980.00	0.00	0.40
23040.00	0.00	0.40

23100.00	0.00	0.39
23160.00	0.00	0.39
23220.00	0.00	0.39
23280.00	0.00	0.39
23340.00	0.00	0.38
23400.00	0.00	0.38
23460.00	0.00	0.38
23520.00	0.00	0.38
23580.00	0.00	0.38
23640.00	0.00	0.37
23700.00	0.00	0.37
23760.00	0.00	0.37
23820.00	0.00	0.37
23880.00	0.00	0.36
23940.00	0.00	0.36
24000.00	0.00	0.36
24060.00	0.00	0.36
24120.00	0.00	0.35
24180.00	0.00	0.35
24240.00	0.00	0.35
24300.00	0.00	0.35
24360.00	0.00	0.35
24420.00	0.00	0.34
24480.00	0.00	0.34
24540.00	0.00	0.34
24600.00	0.00	0.34
24660.00	0.00	0.33
24720.00	0.00	0.33
24780.00	0.00	0.33
24840.00	0.00	0.33
24900.00	0.00	0.33
24960.00	0.00	0.32
25020.00	0.00	0.32
25080.00	0.00	0.32
25140.00	0.00	0.32
25200.00	0.00	0.32
25260.00	0.00	0.32
25320.00	0.00	0.31
25380.00	0.00	0.31
25440.00	0.00	0.31
25500.00	0.00	0.31
25560.00	0.00	0.31
25620.00	0.00	0.30
25680.00	0.00	0.30
25740.00	0.00	0.30
25800.00	0.00	0.30
25860.00	0.00	0.30
25920.00	0.00	0.29
25980.00	0.00	0.29
26040.00	0.00	0.29
26100.00	0.00	0.29
26160.00	0.00	0.29
26220.00	0.00	0.29
26280.00	0.00	0.28
26340.00	0.00	0.28
26400.00	0.00	0.28

26460.00	0.00	0.28
26520.00	0.00	0.28
26580.00	0.00	0.28
26640.00	0.00	0.28
26700.00	0.00	0.27
26760.00	0.00	0.27
26820.00	0.00	0.27
26880.00	0.00	0.27
26940.00	0.00	0.27
27000.00	0.00	0.27
27060.00	0.00	0.26
27120.00	0.00	0.26
27180.00	0.00	0.26
27240.00	0.00	0.26
27300.00	0.00	0.26
27360.00	0.00	0.26
27420.00	0.00	0.26
27480.00	0.00	0.25
27540.00	0.00	0.25
27600.00	0.00	0.25
27660.00	0.00	0.25
27720.00	0.00	0.25
27780.00	0.00	0.25
27840.00	0.00	0.25
27900.00	0.00	0.24
27960.00	0.00	0.24
28020.00	0.00	0.24
28080.00	0.00	0.24
28140.00	0.00	0.24
28200.00	0.00	0.24
28260.00	0.00	0.24
28320.00	0.00	0.23
28380.00	0.00	0.23
28440.00	0.00	0.23
28500.00	0.00	0.23
28560.00	0.00	0.23
28620.00	0.00	0.23
28680.00	0.00	0.23
28740.00	0.00	0.23
28800.00	0.00	0.22
28860.00	0.00	0.22
28920.00	0.00	0.22
28980.00	0.00	0.22
29040.00	0.00	0.22
29100.00	0.00	0.22
29160.00	0.00	0.22
29220.00	0.00	0.22
29280.00	0.00	0.22
29340.00	0.00	0.21
29400.00	0.00	0.21
29460.00	0.00	0.21
29520.00	0.00	0.21
29580.00	0.00	0.21
29640.00	0.00	0.21
29700.00	0.00	0.21
29760.00	0.00	0.21

29820.00	0.00	0.21
29880.00	0.00	0.20
29940.00	0.00	0.20
30000.00	0.00	0.20
30060.00	0.00	0.20
30120.00	0.00	0.20
30180.00	0.00	0.20
30240.00	0.00	0.20
30300.00	0.00	0.20
30360.00	0.00	0.20
30420.00	0.00	0.20
30480.00	0.00	0.19
30540.00	0.00	0.19
30600.00	0.00	0.19
30660.00	0.00	0.19
30720.00	0.00	0.19
30780.00	0.00	0.19
30840.00	0.00	0.19
30900.00	0.00	0.19
30960.00	0.00	0.19
31020.00	0.00	0.19
31080.00	0.00	0.18
31140.00	0.00	0.18
31200.00	0.00	0.18
31260.00	0.00	0.18
31320.00	0.00	0.18
31380.00	0.00	0.18
31440.00	0.00	0.18
31500.00	0.00	0.18
31560.00	0.00	0.18
31620.00	0.00	0.18
31680.00	0.00	0.18
31740.00	0.00	0.17
31800.00	0.00	0.17
31860.00	0.00	0.17
31920.00	0.00	0.17
31980.00	0.00	0.17
32040.00	0.00	0.17
32100.00	0.00	0.17
32160.00	0.00	0.17
32220.00	0.00	0.17
32280.00	0.00	0.17
32340.00	0.00	0.17
32400.00	0.00	0.17
32460.00	0.00	0.16
32520.00	0.00	0.16
32580.00	0.00	0.16
32640.00	0.00	0.16
32700.00	0.00	0.16
32760.00	0.00	0.16
32820.00	0.00	0.16
32880.00	0.00	0.16
32940.00	0.00	0.16
33000.00	0.00	0.16
33060.00	0.00	0.16
33120.00	0.00	0.16

```
33180.00        0.00        0.16
33240.00        0.00        0.15
33300.00        0.00        0.15
33360.00        0.00        0.15
33420.00        0.00        0.15
33480.00        0.00        0.15
33540.00        0.00        0.15
33600.00        0.00        0.15
33660.00        0.00        0.15
33720.00        0.00        0.15
33780.00        0.00        0.15
33840.00        0.00        0.15
33900.00        0.00        0.15
33960.00        0.00        0.15
34020.00        0.00        0.15
34080.00        0.00        0.14
34140.00        0.00        0.14
34200.00        0.00        0.14
34260.00        0.00        0.14
34320.00        0.00        0.14
34380.00        0.00        0.14
34440.00        0.00        0.14
34500.00        0.00        0.14
34560.00        0.00        0.14
34620.00        0.00        0.14
34680.00        0.00        0.14
34740.00        0.00        0.14
34800.00        0.00        0.14
34860.00        0.00        0.14
34920.00        0.00        0.14
34980.00        0.00        0.14
35040.00        0.00        0.13
35100.00        0.00        0.13
35160.00        0.00        0.13
35220.00        0.00        0.13
35280.00        0.00        0.13
35340.00        0.00        0.13
35400.00        0.00        0.13
35460.00        0.00        0.13
35520.00        0.00        0.13
35580.00        0.00        0.13
35640.00        0.00        0.13
35700.00        0.00        0.13
35760.00        0.00        0.13
35820.00        0.00        0.13
35880.00        0.00        0.13
35940.00        0.00        0.13
36000.00        0.00        0.13
```

Listagem 4: `rout02.py` — Resultado numérico do programa `rout02.py` (arquivo `rout02.out`)

```
Tempo (s) I(t) (m3/s) O(t) (m3/s)
        0        0.00        0.00
       60        0.17        0.00
```

120	0.33	0.00
180	0.50	0.00
240	0.67	0.00
300	0.83	0.00
360	1.00	0.00
420	1.17	0.00
480	1.33	0.01
540	1.50	0.01
600	1.67	0.01
660	1.83	0.01
720	2.00	0.02
780	2.17	0.02
840	2.33	0.03
900	2.50	0.04
960	2.67	0.04
1020	2.83	0.05
1080	3.00	0.06
1140	3.17	0.07
1200	3.33	0.09
1260	3.50	0.10
1320	3.67	0.11
1380	3.83	0.13
1440	4.00	0.15
1500	4.17	0.17
1560	4.33	0.19
1620	4.50	0.21
1680	4.67	0.23
1740	4.83	0.26
1800	5.00	0.28
1860	5.17	0.31
1920	5.33	0.34
1980	5.50	0.37
2040	5.67	0.41
2100	5.83	0.44
2160	6.00	0.48
2220	6.17	0.52
2280	6.33	0.56
2340	6.50	0.60
2400	6.67	0.65
2460	6.83	0.69
2520	7.00	0.74
2580	7.17	0.79
2640	7.33	0.85
2700	7.50	0.90
2760	7.67	0.96
2820	7.83	1.02
2880	8.00	1.08
2940	8.17	1.15
3000	8.33	1.21
3060	8.50	1.28
3120	8.67	1.35
3180	8.83	1.42
3240	9.00	1.50
3300	9.17	1.58
3360	9.33	1.66
3420	9.50	1.74

3480	9.67	1.82
3540	9.83	1.91
3600	10.00	2.00
3660	9.89	2.08
3720	9.78	2.17
3780	9.67	2.26
3840	9.56	2.34
3900	9.44	2.43
3960	9.33	2.51
4020	9.22	2.59
4080	9.11	2.67
4140	9.00	2.75
4200	8.89	2.83
4260	8.78	2.90
4320	8.67	2.98
4380	8.56	3.05
4440	8.44	3.12
4500	8.33	3.19
4560	8.22	3.25
4620	8.11	3.32
4680	8.00	3.38
4740	7.89	3.44
4800	7.78	3.50
4860	7.67	3.55
4920	7.56	3.61
4980	7.44	3.66
5040	7.33	3.71
5100	7.22	3.76
5160	7.11	3.81
5220	7.00	3.85
5280	6.89	3.89
5340	6.78	3.94
5400	6.67	3.97
5460	6.56	4.01
5520	6.44	4.05
5580	6.33	4.08
5640	6.22	4.11
5700	6.11	4.14
5760	6.00	4.17
5820	5.89	4.19
5880	5.78	4.21
5940	5.67	4.24
6000	5.56	4.26
6060	5.44	4.27
6120	5.33	4.29
6180	5.22	4.30
6240	5.11	4.32
6300	5.00	4.33
6360	4.89	4.34
6420	4.78	4.34
6480	4.67	4.35
6540	4.56	4.35
6600	4.44	4.35
6660	4.33	4.35
6720	4.22	4.35
6780	4.11	4.35

6840	4.00	4.35
6900	3.89	4.34
6960	3.78	4.33
7020	3.67	4.32
7080	3.56	4.31
7140	3.44	4.30
7200	3.33	4.29
7260	3.22	4.27
7320	3.11	4.26
7380	3.00	4.24
7440	2.89	4.22
7500	2.78	4.20
7560	2.67	4.18
7620	2.56	4.16
7680	2.44	4.14
7740	2.33	4.11
7800	2.22	4.09
7860	2.11	4.06
7920	2.00	4.03
7980	1.89	4.00
8040	1.78	3.97
8100	1.67	3.94
8160	1.56	3.91
8220	1.44	3.88
8280	1.33	3.84
8340	1.22	3.81
8400	1.11	3.77
8460	1.00	3.73
8520	0.89	3.69
8580	0.78	3.66
8640	0.67	3.62
8700	0.56	3.58
8760	0.44	3.53
8820	0.33	3.49
8880	0.22	3.45
8940	0.11	3.41
9000	0.00	3.36
9060	0.00	3.32
9120	0.00	3.27
9180	0.00	3.23
9240	0.00	3.19
9300	0.00	3.15
9360	0.00	3.11
9420	0.00	3.07
9480	0.00	3.03
9540	0.00	2.99
9600	0.00	2.95
9660	0.00	2.91
9720	0.00	2.88
9780	0.00	2.84
9840	0.00	2.81
9900	0.00	2.77
9960	0.00	2.74
10020	0.00	2.70
10080	0.00	2.67
10140	0.00	2.64

10200	0.00	2.61
10260	0.00	2.57
10320	0.00	2.54
10380	0.00	2.51
10440	0.00	2.48
10500	0.00	2.45
10560	0.00	2.42
10620	0.00	2.40
10680	0.00	2.37
10740	0.00	2.34
10800	0.00	2.31
10860	0.00	2.29
10920	0.00	2.26
10980	0.00	2.23
11040	0.00	2.21
11100	0.00	2.18
11160	0.00	2.16
11220	0.00	2.13
11280	0.00	2.11
11340	0.00	2.09
11400	0.00	2.06
11460	0.00	2.04
11520	0.00	2.02
11580	0.00	1.99
11640	0.00	1.97
11700	0.00	1.95
11760	0.00	1.93
11820	0.00	1.91
11880	0.00	1.89
11940	0.00	1.87
12000	0.00	1.85
12060	0.00	1.83
12120	0.00	1.81
12180	0.00	1.79
12240	0.00	1.77
12300	0.00	1.75
12360	0.00	1.73
12420	0.00	1.71
12480	0.00	1.69
12540	0.00	1.68
12600	0.00	1.66
12660	0.00	1.64
12720	0.00	1.62
12780	0.00	1.61
12840	0.00	1.59
12900	0.00	1.57
12960	0.00	1.56
13020	0.00	1.54
13080	0.00	1.53
13140	0.00	1.51
13200	0.00	1.50
13260	0.00	1.48
13320	0.00	1.47
13380	0.00	1.45
13440	0.00	1.44
13500	0.00	1.42

13560	0.00	1.41
13620	0.00	1.40
13680	0.00	1.38
13740	0.00	1.37
13800	0.00	1.35
13860	0.00	1.34
13920	0.00	1.33
13980	0.00	1.32
14040	0.00	1.30
14100	0.00	1.29
14160	0.00	1.28
14220	0.00	1.27
14280	0.00	1.25
14340	0.00	1.24
14400	0.00	1.23
14460	0.00	1.22
14520	0.00	1.21
14580	0.00	1.20
14640	0.00	1.18
14700	0.00	1.17
14760	0.00	1.16
14820	0.00	1.15
14880	0.00	1.14
14940	0.00	1.13
15000	0.00	1.12
15060	0.00	1.11
15120	0.00	1.10
15180	0.00	1.09
15240	0.00	1.08
15300	0.00	1.07
15360	0.00	1.06
15420	0.00	1.05
15480	0.00	1.04
15540	0.00	1.03
15600	0.00	1.02
15660	0.00	1.01
15720	0.00	1.01
15780	0.00	1.00
15840	0.00	0.99
15900	0.00	0.98
15960	0.00	0.97
16020	0.00	0.96
16080	0.00	0.95
16140	0.00	0.95
16200	0.00	0.94
16260	0.00	0.93
16320	0.00	0.92
16380	0.00	0.91
16440	0.00	0.91
16500	0.00	0.90
16560	0.00	0.89
16620	0.00	0.88
16680	0.00	0.87
16740	0.00	0.87
16800	0.00	0.86
16860	0.00	0.85

16920	0.00	0.85
16980	0.00	0.84
17040	0.00	0.83
17100	0.00	0.82
17160	0.00	0.82
17220	0.00	0.81
17280	0.00	0.80
17340	0.00	0.80
17400	0.00	0.79
17460	0.00	0.79
17520	0.00	0.78
17580	0.00	0.77
17640	0.00	0.77
17700	0.00	0.76
17760	0.00	0.75
17820	0.00	0.75
17880	0.00	0.74
17940	0.00	0.74
18000	0.00	0.73
18060	0.00	0.72
18120	0.00	0.72
18180	0.00	0.71
18240	0.00	0.71
18300	0.00	0.70
18360	0.00	0.70
18420	0.00	0.69
18480	0.00	0.69
18540	0.00	0.68
18600	0.00	0.67
18660	0.00	0.67
18720	0.00	0.66
18780	0.00	0.66
18840	0.00	0.65
18900	0.00	0.65
18960	0.00	0.64
19020	0.00	0.64
19080	0.00	0.63
19140	0.00	0.63
19200	0.00	0.63
19260	0.00	0.62
19320	0.00	0.62
19380	0.00	0.61
19440	0.00	0.61
19500	0.00	0.60
19560	0.00	0.60
19620	0.00	0.59
19680	0.00	0.59
19740	0.00	0.58
19800	0.00	0.58
19860	0.00	0.58
19920	0.00	0.57
19980	0.00	0.57
20040	0.00	0.56
20100	0.00	0.56
20160	0.00	0.56
20220	0.00	0.55

20280	0.00	0.55
20340	0.00	0.54
20400	0.00	0.54
20460	0.00	0.54
20520	0.00	0.53
20580	0.00	0.53
20640	0.00	0.52
20700	0.00	0.52
20760	0.00	0.52
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21300	0.00	0.48
21360	0.00	0.48
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21780	0.00	0.46
21840	0.00	0.46
21900	0.00	0.45
21960	0.00	0.45
22020	0.00	0.45
22080	0.00	0.44
22140	0.00	0.44
22200	0.00	0.44
22260	0.00	0.43
22320	0.00	0.43
22380	0.00	0.43
22440	0.00	0.43
22500	0.00	0.42
22560	0.00	0.42
22620	0.00	0.42
22680	0.00	0.41
22740	0.00	0.41
22800	0.00	0.41
22860	0.00	0.41
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23220	0.00	0.39
23280	0.00	0.39
23340	0.00	0.39
23400	0.00	0.38
23460	0.00	0.38
23520	0.00	0.38
23580	0.00	0.38

23640	0.00	0.37
23700	0.00	0.37
23760	0.00	0.37
23820	0.00	0.37
23880	0.00	0.36
23940	0.00	0.36
24000	0.00	0.36
24060	0.00	0.36
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24180	0.00	0.35
24240	0.00	0.35
24300	0.00	0.35
24360	0.00	0.35
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25020	0.00	0.32
25080	0.00	0.32
25140	0.00	0.32
25200	0.00	0.32
25260	0.00	0.32
25320	0.00	0.31
25380	0.00	0.31
25440	0.00	0.31
25500	0.00	0.31
25560	0.00	0.31
25620	0.00	0.31
25680	0.00	0.30
25740	0.00	0.30
25800	0.00	0.30
25860	0.00	0.30
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26220	0.00	0.29
26280	0.00	0.29
26340	0.00	0.28
26400	0.00	0.28
26460	0.00	0.28
26520	0.00	0.28
26580	0.00	0.28
26640	0.00	0.28
26700	0.00	0.27
26760	0.00	0.27
26820	0.00	0.27
26880	0.00	0.27
26940	0.00	0.27

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27060	0.00	0.27
27120	0.00	0.26
27180	0.00	0.26
27240	0.00	0.26
27300	0.00	0.26
27360	0.00	0.26
27420	0.00	0.26
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27780	0.00	0.25
27840	0.00	0.25
27900	0.00	0.25
27960	0.00	0.24
28020	0.00	0.24
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28500	0.00	0.23
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28980	0.00	0.22
29040	0.00	0.22
29100	0.00	0.22
29160	0.00	0.22
29220	0.00	0.22
29280	0.00	0.22
29340	0.00	0.22
29400	0.00	0.21
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29580	0.00	0.21
29640	0.00	0.21
29700	0.00	0.21
29760	0.00	0.21
29820	0.00	0.21
29880	0.00	0.21
29940	0.00	0.20
30000	0.00	0.20
30060	0.00	0.20
30120	0.00	0.20
30180	0.00	0.20
30240	0.00	0.20
30300	0.00	0.20

30360	0.00	0.20
30420	0.00	0.20
30480	0.00	0.19
30540	0.00	0.19
30600	0.00	0.19
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30780	0.00	0.19
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31080	0.00	0.19
31140	0.00	0.18
31200	0.00	0.18
31260	0.00	0.18
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31380	0.00	0.18
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31920	0.00	0.17
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32280	0.00	0.17
32340	0.00	0.17
32400	0.00	0.17
32460	0.00	0.17
32520	0.00	0.16
32580	0.00	0.16
32640	0.00	0.16
32700	0.00	0.16
32760	0.00	0.16
32820	0.00	0.16
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33060	0.00	0.16
33120	0.00	0.16
33180	0.00	0.16
33240	0.00	0.16
33300	0.00	0.15
33360	0.00	0.15
33420	0.00	0.15
33480	0.00	0.15
33540	0.00	0.15
33600	0.00	0.15
33660	0.00	0.15

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33780        0.00        0.15
33840        0.00        0.15
33900        0.00        0.15
33960        0.00        0.15
34020        0.00        0.15
34080        0.00        0.15
34140        0.00        0.14
34200        0.00        0.14
34260        0.00        0.14
34320        0.00        0.14
34380        0.00        0.14
34440        0.00        0.14
34500        0.00        0.14
34560        0.00        0.14
34620        0.00        0.14
34680        0.00        0.14
34740        0.00        0.14
34800        0.00        0.14
34860        0.00        0.14
34920        0.00        0.14
34980        0.00        0.14
35040        0.00        0.14
35100        0.00        0.13
35160        0.00        0.13
35220        0.00        0.13
35280        0.00        0.13
35340        0.00        0.13
35400        0.00        0.13
35460        0.00        0.13
35520        0.00        0.13
35580        0.00        0.13
35640        0.00        0.13
35700        0.00        0.13
35760        0.00        0.13
35820        0.00        0.13
35880        0.00        0.13
35940        0.00        0.13
36000        0.00        0.13
```

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